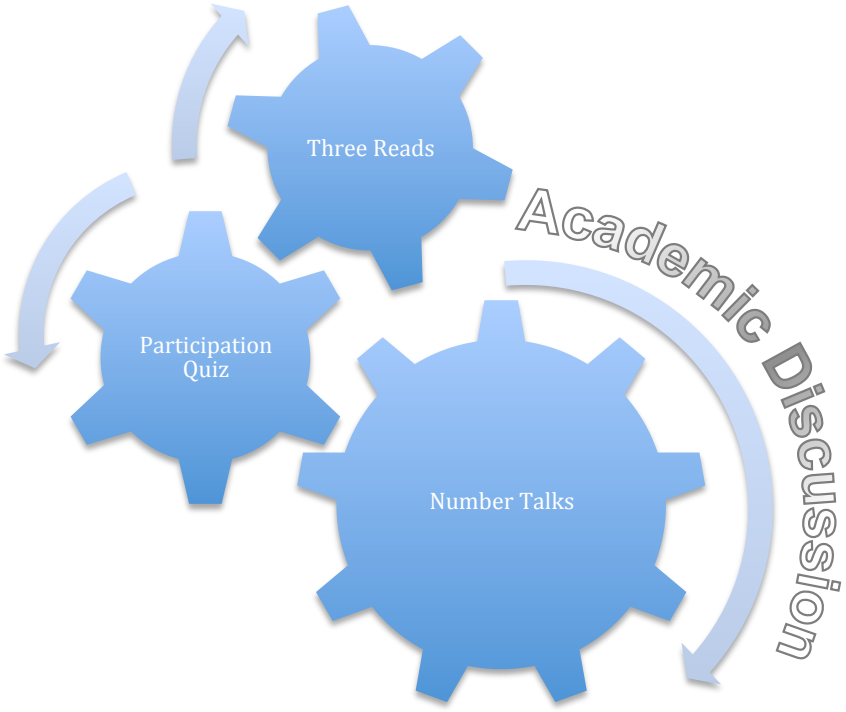

Instructional Toolkit for Mathematics

Strategies and experiences that define what it means to learn and teach mathematics in Oakland

Oakland Unified School District
Leadership, Curriculum and Instruction

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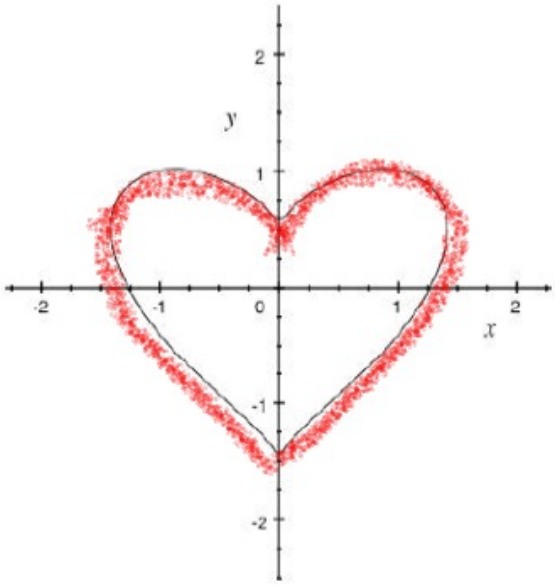

math

"The greatest impact on learning is the daily lived experiences of students in classrooms, and that is determined much more by how teachers teach than by what they teach."

—Dylan William (2011)

Why an Instructional Toolkit?

The Leadership, Curriculum & Instruction (LCI) Mathematics team recognizes that strong classroom practices are at the core of meaningful learning experiences for students. This Instructional Toolkit, and the specific signature strategies it contains, aims to breathe life into the curriculum and to describe a vision for math pedagogy that characterizes the experience of mathematics teaching and learning throughout OUSD. Our hope is that by using the strategies found in this toolkit, teachers and students will get maximum benefit from any lesson at every grade level.

We have found that the three key strategies presented here provide students with the tools and opportunities they need to develop as mathematical thinkers and problem solvers, and to recognize that mathematics, at its core, makes sense. These math-specific strategies support the broader focus on Academic Discussion as OUSD's signature pedagogy, which connects all content areas. The Instructional Toolkit for Mathematics is aligned with a district-wide, cross-content effort to create a cohesive experience for students throughout their school journey.

This Toolkit is the product of a collaborative effort between the LCI Mathematics team, content experts in the mathematics field, and teachers from across the district. We are particularly grateful to the many teachers who have tried some of the different strategies in their classrooms over the last year and who have shared their insights and observations about how the strategies support student learning.

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"Dialogue is the central aspect of co-intelligence. We can only generate higher levels of intelligence among us if we are doing some high quality talking with each other."

—Tom Alec, The Tao of Democracy

District-wide shift

By 2016, Oakland Unified School District will be in the final stage of the OUSD Strategic Plan to establish sustainable Effective Full Service Community Schools district-wide. The vision is made real when every school in OUSD has integrated academic learning with essential supports and opportunities, and site programs are structured around a high quality instructional core to create equitable opportunities for learning. Central to these goals are students that experience rigorous and effective instruction every day.

Coincidentally, by 2016 California will have fully implemented the Common Core State Standards in English Language Arts & Literacy and Mathematics (adopted by the State Board of Education in 2010). The Next Generation Science Standards are expected to be adopted by California in 2014. These concurrent curricular changes are accompanied by new standards for the student learning experience supporting college, career and community readiness.

"More important than the topics covered, or even the skills used directly in class, are the more general abilities and attitudes that should be gained in the effort of mastering the content."

— University of California A-G Guide (c) Mathematics

How can students simultaneously increase their content knowledge and develop their academic character? They will need as many opportunities as possible to practice, revise, and demonstrate these skills. Effective instructional routines that emphasize learning through discourse will engage students, meet their individual needs, and help them develop as learners of mathematics and across all disciplines. OUSD envisions that every math classroom provides daily opportunities for students to demonstrate the Standards for Mathematical Practice:

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

In order to create a learning community that provides students with a variety of means to develop their mathematical character, teachers must increase the quality and quantity of academic discussions. This Instructional Toolkit contains structures and strategies that elicit and develop the Standards for Mathematical Practice for all students.

"It's not about right answers. It's about developing reasoning and the capacity to articulate and communicate."

—Lucy West, Metamorphosis Teaching Learning Communities

The Graduate Profile

Oakland Unified recognizes that there are many paths for students to follow upon completion of high school, and the district's charge is to graduate each individual with the content knowledge and social and academic character necessary to be able to pursue his or her chosen course. The Graduate Profile describes the core elements of the education Oakland Unified School District strives to provide for every one of its students, as well as the competencies of an OUSD graduate.

Oakland Unified School District Graduate Profile

"Career is the goal - Education is the path"

"Our graduates are college, career, and community ready!"

The Student Experience:

A Culture of Discourse

Quality Academic Discussions Across the District

(adapted from OUSD's Quality Academic Discussion website)

Oakland Unified has made a decision to focus on Academic Discussion as a signature pedagogy across all content areas, grades PK-12. The following offers an overview of the multiple aspects of Academic Discussion, and the different structures where it is emphasized.

Quality academic discussions:

- are purposeful, sustained conversations about content which require students to work together to co-construct knowledge and negotiate meaning in order to attain high levels of thinking and deep understanding about a topic
- are characterized by high student engagement and participation, as students discuss relevant content that allows for multiple and diverse perspectives

Quality academic discussions in mathematics:

- Allow students to make sense together and reason logically about mathematics.
- Motivate students to pay attention to their peers and to articulate their own ideas thoughtfully
- Cause student misconceptions to surface, and allow teachers a means by which to understand student thinking and address those misconceptions in meaningful ways.

There are **whole-class**, **small group**, and **partner structures** for academic discussions, each of which has important and distinct purposes and benefits for all learners, and each of which offers strong support for English Language Learners. These structures immerse students in the Mathematical Practices identified by Common Core as essential to students' math development

Whole-class discussion is an opportunity for the teacher to facilitate academic discussions in the classroom by questioning students in such a way that helps them articulate their thinking and build off each others' ideas. Teachers may also use this time to model and monitor the language and discussion skills students are expected to apply in small groups. This is particularly helpful for English learners who benefit greatly from hearing correct pronunciation and usage of new language. Whole-class discussion can be used to synthesize and deepen student thinking and build common understanding after a learning experience or small group discussion. Whole class discussion can also be used to assess understanding and to surface and address misconceptions about content.

Small group discussion provides students an opportunity to develop and apply discussion skills in a less controlled environment. In mathematics, small groups can maximize the opportunity for each student to talk, explore mathematical notions, articulate reasoning, and develop a positive mathematical identity. Additionally, in preparation for a rich whole-class discussion, small group discussions can provide a comfortable space for students to rehearse ideas before moving to the whole class structure. We define small groups as 3-5 students.

Partner structures provide maximum speaking time because they enable half of the students in a class to speak at any one time. Compared to whole-class and small group discussion, there is often greater accountability in a partner discussion. For example, students in a partner discussion are positioned to listen attentively and work to understand what their partner says since they are the sole person expected to respond. A partner discussion also provides a safe and comfortable environment (as opposed to a whole-class setting) to try out new language, new understanding, and new ideas, which can be especially important for English learners and for students who are reluctant to step forward in larger settings.

Each of the key strategies described in this Toolkit suggests teacher moves that support active student participation in one or more of the discussion structures described above.

"If we use talk to establish a supportive learning environment, but we do not succeed in creating productive talk about the actual content of mathematics, our students are not likely to succeed in learning the mathematics we want them to know. On the other hand, if we fail to build a supportive learning environment—one in which students can talk about their mathematical thinking without fear of ridicule—then many students will not participate at all."

—Chapin, O'Connor, and Anderson (2009)

Social and Emotional Learning

Developing the Standards for Mathematical Practice and growing toward the OUSD Graduate Profile require individuals to increase their competence in Social and Emotional Learning. Social and Emotional Learning (SEL) is a process for helping children and adults develop the fundamental skills for life effectiveness. SEL skills are the skills all individuals need to manage themselves, their relationships, and their work effectively and ethically. The Common Core State Standards push students to construct arguments, critique the reasoning of one another, and persevere in solving complex problems. SEL skills are foundational to students' and teachers' ability to engage in those practices in respectful and productive ways. These skills help to establish the conditions for learning that make the graduate profile achievable, and allow classroom communities to meet the rigor of the Common Core State Standards.

OUSD Focus Areas:	**Essential SEL Skills for Mathematics Include:**
Speaking and listening in academic discussions Reading increasingly complex texts with comprehension Writing arguments with evidence	➢ The ability to extend and build upon one another's answers ➢ Listening to one another's explanations ➢ Making sense of and persevering in solving complex problems ➢ Asking thoughtful questions to clarify or improve the arguments of others ➢ Expressing divergent points of view while peers explore the truth of their position

Support for building SEL capacity in children, youth and adults is available through the OUSD Office of Social and Emotional Learning and Leadership.

For additional information and support, including resources, tools, videos and research, visit: https://sites.google.com/a/ousd.k12.ca.us/socialemotionallearning/

Standards for Mathematical Practice

The Common Core Standards for Mathematics come in two parts—the **Content Standards** which describe what is taught in each grade or course, and the **Standards for Mathematical Practice** which describe eight areas of expertise that students should develop over the course of their school math careers:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Evidence Gathering Tool: the Math 5"×8" Card

OUSD and our Mathematics Partners have translated the Standards for Mathematical Practice into an observable set of student behaviors, or "Student Vital Actions". These Student Vital Actions are described on the evidence gathering tool shown below, known as the 5x8 card, which is used to focus observations of students doing math and to make the Standards for Mathematical Practice visible in the classroom.

5x8 Evidence-Gathering Card

Principle	Student Vital Actions
Logic connects sentences *Practices 1, 2, 3, 6*	Students ***say a second sentence*** (spontaneously or prompted by the teacher or another student) to explain their thinking and connect it to their first sentence.
Reasoning develops when students develop viable arguments *Practices 1, 2, 3, 6, 7, 8*	Students ***talk about each other's thinking*** (not just their own).
Students write explanations *Practices 1 - 8*	Students **write their mathematics**, and connect multiple representations of their thinking (e.g. pictures, diagrams, numbers, words, tables, graphs, expressions, etc.). Students revise their thinking, and their written work includes ***revised explanations*** and justifications.
Academic success depends on academic language *Practices 3, 6*	Students use ***general and discipline-specific academic language*** in their oral and written explanations and discussions (spontaneously and/or prompted by the teacher or other students.)
ELLs develop language through content	***English learners produce language*** that communicates ideas and reasoning, even when that language is imperfect. They take advantage of available language supports and resources: peer support, sentence frames, multiple choice oral responses, visual representation, graphic organizers, home language, cognates, etc.
A growth mindset matters	***Interview- Do students believe that they can learn to be good at math*** by learning more math, by working hard, and persevering to make sense of problems? Or do students think they cannot change how good at math they are?
Equity (The foundation for the above)	***Which students are participating***? (e.g. boys more than girls, the same few students, ELL and special ed students?) Are they volunteering? Called on to do math? Talking about math in their group? Off task? **All students ask math questions.**

OAKLAND UNIFIED SCHOOL DISTRICT
Community Schools, Thriving Students

V.8 1-15-13
Your input welcome: TeamMath@ousd.k12.ca.us

"Through classroom discourse of certain types, students are encouraged to treat one another as equal partners in thinking, conjecturing, exploring, and sharing ideas. Mutual respect is fostered when norms exist that set expectations for respectful and civil discourse."

—Chapin, O'Connor, and Anderson (2009)

Socio-Mathematical Norms

Most classrooms have a set of norms and agreements shared by teachers and students that help establish the classroom culture.

The Socio-Mathematical Norms below articulate beliefs held in OUSD about learning math. The norms name specific expectations and conditions that support learning in a context of academic discussion. While specific to mathematics, the norms can easily apply to other content areas, and can serve as a complement other existing classroom norms and agreements.

Socio-Mathematical Norms Promote Learning

Errors are gifts, because they promote discussion.

The answer is important, but it is *not* the math.

Ask questions until ideas make sense.

Share a second sentence to connect your thoughts.

Talk about each other's thinking.

Think with language and use language to think.

Use multiple strategies … multiple representations!

Instructional Practice that Brings the Student Experience to Life:

Three Key Strategies for Oakland Unified Math Classrooms

Three Signature Instructional Strategies

As Oakland math classrooms transition to the Common Core State Standards in math (CCSS-M), students are engaging in more non-routine tasks and are participating in mathematical discourse. They are learning to persevere in the face of challenging mathematics, and to recognize productive struggle as key to deep learning. For teachers, the transition to the CCSS-M can be summarized in three shifts in practice: the use of non-routine tasks as the context for learning mathematics content, increased discourse in the classroom, and more opportunities for students to engage in productive struggle to make meaning of mathematics.

The district focus on academic discussion as an overarching pedagogy across all content areas supports each of these shifts. More specifically, the math department has identified three key instructional strategies that fall within the pedagogy and allow teachers to bring those shifts to life in their classrooms.

Each of the strategies speaks to a specific feature of a student's experience in math classrooms that helps to define what it means to learn and to teach mathematics in Oakland. As the strategies become common tools for teachers in Oakland classrooms, they will help to infuse quality academic discussion in all aspects of the school, contributing to a cohesive experience for students as they move through the grades and through different mathematics courses.

In this section, you will find information about the following strategies:

Number Talk. *A strategy to build flexibility, accuracy and efficiency in mathematical thinking through the articulation of and sharing of mental math strategies.*

3-Read. *A strategy for engaging students in making sense of a problem before they set out to solve it.*

Participation Quiz. *A strategy to support productive small group work on mathematics.*

Each strategy is described in general, with information about when the strategy is useful, why a teacher might choose to use it in the context of a specific lesson, and what students will learn from the experience. That overview is followed by:

- Specific, step-by-step description of how the strategy comes to life in a classroom.
- Guidance for identifying a math problem or task to pair with the strategy
- A planning tool template
- Samples of the planning tool completed for a specific problem or task
- Additional information, suggestions, and tips

Number Talk

Purpose: flexibility, accuracy and efficiency in mathematical thinking through the articulation of and sharing of mental math strategies

GENERAL DESCRIPTION:

A **Number Talk** is a 10 to 15 minute whole group *mental math* activity where students find the answer to a math problem in their heads, then share aloud the strategies they used to find that answer. This strategy helps to develop quality student discourse in a whole class setting as students are encouraged to explain their thinking, justify their reasoning, and make sense of each other's strategies.

During a Number Talk, the teacher steps away from his/her role of authority, and into the role of facilitator by asking students questions, recording student responses on the board, and encouraging students to make meaning out of the mathematics through verbal exchange.

A Number Talk can be used to address gaps in student skills or understanding, to confront anticipated misconceptions, to surface multiple strategies, and/or as a formative assessment when introducing new concepts. Number Talks also build flexibility, accuracy and efficiency with numbers for all students.

In lower elementary, students might experience a Number Talk where they have to look at a pattern of dots for 3 seconds, and share strategies for how they knew the total number of dots. In upper elementary, students may be asked to multiply 25×8 and may use different decomposition strategies or their knowledge of money to calculate. In middle school, students may be asked to mentally find 35% of 160. In high school, they may share multiple strategies for solving $125^{2/3} \cdot 2$. Number Talks may be used to make sense of grade-level content, but can also build from concepts from previous classes by starting with a dot talk or a simple arithmetic problem at any grade, based on where the students' needs are.

WHEN AND WHY IS THIS USEFUL?	WHAT CAN STUDENTS LEARN FROM THIS EXPERIENCE?
A Number Talk is useful: • To help students move from a reliance on memorization to truly understanding numbers and their relationships to each other. • To help students recognize structure, and use that structure to understand more complex mathematics. • As a regular routine where the problems in a series of Number Talks build on each other. • To launch a task by activating students' prior knowledge. • To provide students the opportunity to practice explaining their thinking and asking each other questions. • To develop a stronger sense of mathematical identity and self-confidence in students, since mistakes are treated as learning opportunities, and everyone's opinion contributes to group knowledge.	• Flexibility, accuracy and efficiency with mathematical thinking • Ways to make sense of the mathematics and talk about the strategies used to solve a problem. • Ease with composing and decomposing numbers • Conceptual understanding of the relationships between numbers • Computation strategies • Mathematical reasoning skills • Precision in explanations of mathematical thinking • Multiple strategies and multiple representations for finding an answer • Learning from the ideas of peers • Confidence and motivation, contributing to a positive mathematical identity • Empowerment through validation of each person's mathematical thinking process • The value of both successes and errors in deepening understanding

Number Talk: Step-by-Step

Ready to try a Number Talk? Here are step-by-step instructions for structuring a Number Talk in a lesson.

1. Start by identifying the mathematics students will need to be successful in a curricular unit. Are there basic mathematical structures or patterns that, if recognized and understood by students, will lead to understanding of more complex mathematics? Would familiarity and flexibility with particular understandings about numbers, or kinds of computations, support students to make connections between the new material and things they have learned previously? Are there basic skills students may lack that might hinder their progress with new material? From this list, write a problem that students can tackle mentally and to which they might apply several different solving strategies. (See "Choosing a Problem," below.)

2. Anticipate the different strategies that students might apply to finding an answer to the question. Consider how to record each of these strategies so that the symbols or diagrams accurately reflect the strategy. This can be one of the most challenging parts of facilitating a Number Talk, but is critical. Scribing in a way that accurately represents students' thinking allows students to see the structure of their thinking and to compare different strategies.

3. Before posing the question to the class, remind students that this is a mental math exercise, and that everyone will have time to arrive at an answer silently before the discussion begins. If they arrive at an answer before the silent time is up, they should try to think of a different strategy for finding the answer. Demonstrate any silent signals you want students to use to indicate when they are ready – such as putting a fist to their chest when they have an answer, and raising a finger for each additional strategy they think of.

4. Surface the different answers students reached. Poll the class to determine if most students got a specific answer. Then, call on students to share strategies and record their solving processes. As much as possible, based on the complexity of the problem and the strategy, listen through a student's full explanation before scribing, so that the strategy can be accurately represented and you can avoid assuming or prompting a next step by how initial numbers are recorded.

5. Wrap up the number talk. Closure can be achieved, through a discussion, such as identifying similarities and differences between strategies, or by connecting the number talk to the material of the unit, or by asking students to apply a strategy different from their own to a new problem.

Choosing a Problem

A Number Talk may stand alone in a unit as a way to activate specific prior knowledge or introduce new content, or Number Talks may be organized in a series over several lessons in order to point to particular structures in numbers and expressions.

The problem that a Number Talk is organized around may take different forms. Examples include:

Solve or evaluate an expression:	$53+37=?$ "Simplify $\frac{50000}{150}$"
Compare two values:	"Determine which is greater: $\frac{2}{3}$ or $\frac{10}{18}$"
Consider a string of computations to identify patterns:	$2\times30=60$ $3\times30=90$ $4\times30=120$ $5\times30=150$ $6\times30=?$
How many dots are there? How did you see the dots?	

When crafting a problem as the focus of a number talk, consider:

- Does the problem lend itself to mental math? Numbers should be friendly enough to manipulate without pencil and paper. If multiple steps are required, the numbers should be easy to retain as students process through the math.
- Are there multiple strategies for solving the problem? Some problems may be able to be visualized in multiple ways (geometrically, on a number line, on a graph, with an expression, using friendly numbers). Numbers may be able to be composed or decomposed in different ways to achieve an answer. If you cannot readily find two or three different ways to arrive at an answer, the problem may not stimulate multiple strategies or discussion from students.
- If the Number Talk falls in a series, does it allow students to build on the strategies used in the previous Number Talk? Do multiple strategies translate to the new problem? Are all of the strategies applicable in the new context, and is there a value to reasoning about why or why not?

Number Talk – Defining Features

The chart below outlines some of the core features of a Number Talk that distinguish it from any other mathematical discussion about a problem.

Facilitation Feature	Student Experience
Problems are written and read publicly, but students solve mentally (no pencil and paper or white boards)	• Students develop efficiency, accuracy and fluency with mathematical thinking using mental math. • Students move away from a reliance on standard algorithms and strict memorization, and move into sense-making and sharing their reasoning around the mathematics.
Wait time	• All students have time to reflect upon and struggle with mental math and/or come up with multiple ways of solving
Silent signals as mode of response ("I have an answer" "I have 2 strategies"…) Silent validation of who got the same answer / who agrees or disagrees with an answer	• Students are not distracted by hands in the air, or by others who have found an answer quickly and want to share immediately. • Students are motivated to come up with more than one way of solving. Emphasis is placed on the thinking process more than the answer itself. • Students interact with each other, not just with the teacher
Surface all answers up front, including mistakes	• Mistakes are treated as learning opportunities • Students agree with and/or critique the reasoning of others
Turn and Talk (optional)	• Every student has an opportunity to share her/his way of thinking about and solving the problem • Students articulate ideas with a partner before engaging in large group academic discussion
Teacher begins scribing /representing student's strategy *after* student has finished explaining and without steering student in a particular direction. Teacher confirms with the presenter that his/her thinking is properly represented.	• Multiple strategies are made public • Students see different ways to record a mental process • Scribing reflects student's actual process, and not a specific, anticipated solution path • Students feel ownership of their own strategies
Engagement /participation /comprehension questions after strategies are shared. • *Who did it exactly the same way as ___? ("raise your hand if…")* • *Can you do that? Is that legal?* • *Did everyone understand ____'s way?* • *Can someone explain ___'s strategy in your own words?* • *Who has another way of solving it?*	• Students make sense of each other's strategies • Students see multiple ways of mentally solving problems, make connections between different ways of solving problems • Students talk about their own and each other's thinking

Number Talk Planning Tools

The following pages include two different blank templates for planning a Number Talk as part of a math lesson. The blank templates are followed by examples of completed templates for two different Number Talks, one for a primary class and one for a secondary class.

The lesson planning templates are intended to guide a teacher's thinking as s/he prepares to facilitate a Number Talk, including anticipating student responses, considering possible ways to scribe different strategies, and identifying questions that will elicit student thinking and prompt students to make sense of each others' ideas. Thinking through each of these steps is important as teachers build familiarity with the strategy.

Anticipating student responses is a particularly crucial step in preparing to facilitate a Number Talk. Because they are developing both their understanding of the mathematics and their ability to articulate their thinking, students will often share strategies that are difficult to understand, either because the reasoning is complicated or because their language is not precise, or both. This can be particularly true for younger students. Anticipating student responses before presenting the problem to the class, and giving thought to the kinds of strategies students might apply to the problem, helps to ensure that the teacher will be able to find the mathematical logic in any student's contribution. Asking questions that encourage students to elaborate, or having other students paraphrase, can also help to reveal more of a student's intention and support the student in articulating his/her thoughts.

Once Number Talks become a routine part of classroom practice with a group of students, the procedures for how students signal that they have an answer in mind, how the Number Talk is framed, and how answers are shared before strategies are discussed may become habits for both teacher and students and will require less formal planning. At that point, teacher planning can focus more narrowly on anticipating responses and identifying connections a teacher hopes students will recognize or specific strategies that a teacher hopes will surface through the discussion.

Number Talk Lesson Planning Template 1: Narrative

Grade Level: _____ **Unit:** _____

Core Math Idea:

Number Talk Problem:

Anticipated student methods and how to represent them:

During the Lesson

Frame for the activity: We are using a Number Talk to share different strategies for how we mentally approach a problem. Each person's role is to work on explaining their own thinking clearly, and to listen to other's explanations as well.

Maximum length of quiet time: _____

Silent signal when students are ready: _____

Process for sharing out:

-
-
-

Questions to orchestrate the class conversation about strategies:

Wrapping Up:

Number Talk Lesson Planning Template 2: Chart

Grade Level: **Unit:**

Core Math Idea:

Number Talk Problem	Possible Strategies & Method of Recording	Questions to Students
Follow up Problems (series)		**Wrap Up**
Academic Language		

Number Talk Lesson Plan 1: Elementary Sample

Grade Level: 3-5 **Unit:** Multiplication and Division

Core Math Idea: Students may be hindered in this unit because they are not yet fluent with basic addition and subtraction facts. So in this Number Talk, I will focus on adding and subtracting single and double digit numbers mentally, and specifically on the idea of **doubles plus/minus one.**

Number Talk Problem(s): 15 + 16 (First in a series, to be followed by 15 +14; 20 + 21; 22 + 23; 22 + 21

Anticipated student methods and how to represent them:

Standard algorithm (stack them in your head)

$$\begin{array}{r} 15 \\ +16 \\ \hline \end{array}$$

Count on fingers: 15, 16, 17,... **31** (use open number line to represent single jumps)

Add 10 then add six

$$15+10=25$$
$$25+6=31$$

Double 15, then add one more:

$$15+15=30$$
$$30+1=31$$

Add 10 and 10, then add 5, then add 6

$$10+10=20$$
$$20+5=25$$
$$25+6=31$$

During the Lesson

Frame for the activity: We are using a Number Talk to share different strategies for how we mentally approach a problem. Each person should be ready to explain their process, and to listen to understand someone else's.

Maximum length of quiet time: 2 min

Silent signal when students are ready: Thumb up in front of your chest when you have an answer. Raise another finger for each different strategy you think of.

Process for sharing out:

- Talk to your partner about your strategy.
- Volunteers, what number did you get for your solution? (Record all responses)
- After sharing, poll the class – raise hand if you got this value

Questions to orchestrate the class conversation about strategies

- Who would like to share how they got their answer?
- I heard you say _____, did I hear correctly?
- Did anyone use a different method?
- Can someone explain _____'s strategy in their own words?
- Please raise your hand if you understand what _____just shared.

Wrapping Up: Questions I might ask:

- Can you find two strategies that are similar? How are they the same?

  Look at all of these strategies. Which new strategy would you want to try to use tomorrow

Number Talk Lesson Plan 2: Elementary Sample

Grade Level: 3-5 **Unit:** Multiplication and Division

Core Math Idea: Adding and subtracting single and double digit numbers mentally, and specifically the idea of ***doubles plus/minus one***.

<table>
<tr><th>Number Talk Problem</th><th>Possible Strategies & Method of Recording</th><th>Questions to Students</th></tr>
<tr><td>15+16</td><td rowspan="5">Standard algorithm (stack in your head):
15
+16

Count on fingers:
15, 16, 17,... 31
(use open number line to represent single jumps)

Add 10 then add six:
15+10=25
25+5=31

Double 15, then add one more:
15+15=30
30+1=31

Add 10 and 10, then add 5, then add 6
10+10=20
20+5=25
25+6=31</td><td>• Who would like to share how they got their answer?
• I heard you say _____, did I hear correctly?
• Did anyone use a different method?
• Can someone explain _____'s strategy in their own words?
• Please raise your hand if you understand what ____just shared.</td></tr>
<tr><th>Follow up Problems (series)</th><th>Wrap Up</th></tr>
<tr><td>15+14
20+21
22+23
22+21</td><td rowspan="3">Can you find two strategies that are similar? How are they the same?

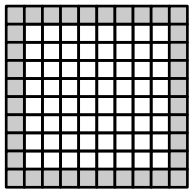
Look at all of these strategies. Which strategy would you want to try to use tomorrow?</td></tr>
<tr><th>Academic Language</th></tr>
<tr><td>addend, sum, total, doubles, doubles plus one, doubles minus one</td></tr>
</table>

Number Talk Lesson Plan 1: Secondary Sample

Grade Level: *5th through 11th* **Unit:** *Equations and Expressions*
Core Math Idea: *Modeling real world situations with expressions, equivalent expressions*

Number Talk Problem:

Given a 10 x 10 grid, what is the area of the border?
(Show students the diagram)



Anticipated Methods and how to represent them:

Saw four strips of 10 around the perimeter but realized that corners were counted twice.	$10+10+10+10-10-1-1-1$ **or** $4(10)-4(1)$
Saw 4 center strips of 8 and added in the corners *How did you know there were 8 in a strip	$4(8)+4(1)$ $4(10-1-1)+4(1)$
Saw two strips of 10, one at the top, one at the bottom, and that left two strips of 8 on the right and left side. *How did you know there were 8 on the right and left?*	$2(10)+2(8)$ $2(10)+2(10-2)$
Saw the strips of 10 on the right and left sides.	$2(10-2)+2(10)$

<u>During the Lesson</u>

Frame for the activity: "We are using a Number Talk to share different strategies for how we mentally approach a problem. Each person's role is to work on explaining their own thinking clearly, and to listen to other's explanations as well."

Maximum length of quiet time: *3 minutes*
Silent signal when students are ready: *Fist to chest when you have an answer. Show on your fingers how many methods you can think of.*

Process for sharing out:

- Turn and Talk about your strategy
- Popcorn out, what number did you get for the area of the border? (Record all responses on board)
- Raise your hand if you got this value.

(continued on next page)

Questions to orchestrate the class conversation about strategies: (10 minutes)

- Who would like to share how they got their solution? (LISTEN, consider how to scribe expression.)
- I heard you say _____, is that correct? (Get affirmation, then SCRIBE.)
- Please raise your hand if you understand what ____ just shared.
- Did anyone use a different method?
- Can someone explain ____'s strategy in their own words? (LISTEN, consider how to scribe expression.)
- _____, did _____ explain your method correctly? (Get affirmation, then SCRIBE.)
- Does someone have another strategy? (LISTEN, consider how to scribe expression. Student may need to approach the diagram to motion through their thinking)
- Can someone please repeat for me what _____ just described so that I can write it down?
- Might there be another method out there? (LISTEN, consider how to scribe expression. Student may need to approach the diagram to motion through their thinking)

Wrapping Up:

Questions I can ask:

- What do you notice about the expressions on the board? (Record full sentence statements)
  Are the expressions equivalent? How do we know? How can we check?
- *If a student has this expression [10 – 1 + 10 – 1 + 10 – 1 + 10 – 1)], what might that tell me about their strategy? What might a student who writes 4(10 – 1) have seen in the problem?
- *Will all of these strategies find the border for any square arrangement?

3-Read

Purpose: engaging students in making sense of a problem before they set out to solve it

GENERAL DESCRIPTION:

3-Read is a mathematics and language comprehension strategy designed to delay the rush to an answer, deepen student understanding of both the situation and the mathematics, and help students make sense of a problem before setting out to solve it. The strategy consists of reading the stem of a problem (the problem without a question) three times aloud, in close proximity, while establishing a specific purpose for each read: 1) comprehending the text; 2) comprehending the mathematics; and 3) eliciting mathematical questions based on the information provided.

Too often, students disengage from math problems, and simply take the numbers and do something with them (add, subtract, multiply or divide). 3-Reads is designed to engage them in making sense of the problem first, and then drawing connections between the situation and the quantities presented. By asking students to come up with mathematical questions on their own, 3-Reads focuses their attention on the context and the mathematical structures, and helps to ensure that students understand both the explicit and the implicit information and quantities presented, setting them up for meaningful productive struggle with a math problem. It delays their need for an immediate answer, and helps students get to the mathematics of a lesson or a unit.

Finally, 3-Reads is a whole group strategy that helps students develop language and mathematical literacy, healthy habits of mind, and an increased capacity to engage in problem solving, all of which build students' capacity for doing mathematics both in a group and individual setting.

WHEN AND WHY IS THIS USEFUL?	WHAT CAN STUDENTS LEARN FROM THIS EXPERIENCE?
3-Read is useful: • For unpacking complex language, and focusing student thinking around understanding the quantities in a problem and their relationships to each other. • For understanding word problems containing explicit and non-explicit quantities, particularly when the problem is not in the student's primary language. • For surfacing the math that helps students get to the mathematics of the unit. • For identifying quantities and context • For delaying the answer-getting, and facilitating student engagement in productive struggle and reasonable problem solving. • For developing mathematical literacy	• Ways to see the mathematical structure of a word problem • How to identify and understand the mathematical structure of the problem in a way that will allow them to generalize to similar word problems • Strategies to draw connections between the context of a situation and its relationship to a math solution • Strategies to help them unpack word problems on their own • Ways that the skills they use with text in other disciplines can support their understanding of math. • The joy of engaging in mathematics that makes sense.

3-Read: Step-by-Step

Ready to try using 3-Read? Here are step-by-step instructions for structuring 3-Read in a lesson.

1. Start by identifying a problem that would be more accessible to students through a 3-Read. Consider whether the problem includes quantities in context, relationships between quantities, and/or nuanced language. (See "Identifying a Problem," below.)
2. Anticipate the points of confusion students may encounter. Identify specific phrases that might be unclear or open to interpretation, and the numerical relationships that are likely to be the greatest obstacle. Before engaging students in the 3-Read, think about everything you want them to extract from the information given. If appropriate, craft follow-up questions for Read 2 (below) to focus students' attention on key parts of the problem.
3. Frame the strategy. Tell students the class will read the problem three times, and that each time you will ask students to answer a specific question. The first time, students will need to listen carefully because they will not have the problem in front of them in writing.
4. Go through each of the three reads, debriefing after each question.

   Read 1: **Comprehending the text.**

   Before showing the text to students, teacher reads the problem (minus the problem stem) out loud, and prompts: *"I want you to think about what is going on in this problem. What is the context of the situation?"* This debrief is not going after the mathematics or the quantities and relationships, but probing how clearly students understand the context so they can make sense of the mathematical situation in the next read.

   Read 2: **Comprehending the mathematics.**

   At this point, students should have the written problem in front of them, either projected or on paper. Teacher or student reads the problem stem again and prompts: *"What are the quantities in the problem? What do they mean? How are they related"* (in early grades, teacher might offer "quantities are numbers or amounts," in later grades teachers might offer, "quantities are numbers and their units"). Students should identify quantities. Those quantities may be explicit (70 inches) or implicit (i.e., if 70 inches and 64 inches are both stated in the stem then the difference of 6 inches is implicit, or if a quantity is referenced without a numerical value, such as "Leo's height").

   Read 3: **Listing all possible mathematical questions.**

   Problem is read, and teacher prompts: *"What are all the different mathematical questions you can think of about this situation?"* The questions students come up with should focus on the quantities presented and the relationships between them. This step should not take significant time; the process of eliciting mathematical questions offers the teacher an opportunity to assess quickly how well students have made sense of the context and structures of the math problem. This debrief ends with, *"... and here is the problem we are going to work on today."*
5. Allow students to work on the problem.

Identifying a Problem

If you anticipate that students may find it difficult to read and interpret a particular problem, using a 3-Read may help students to unpack the text and focus on the core mathematics. However, 3-Read is not an appropriate strategy for every text-rich problem. Here are some things to consider when deciding whether a 3-Read is a strategy that is well-matched to a particular problem.

- **Does the problem have quantities (both implicit and explicit)?** An ideal problem for using 3-Read includes quantities that are easily visible, as well as a quantity that is implicit. For example, it might refer to "10 flowers, four of which are purple and the rest of which are orange." Identifying quantities (Read 2) points attention to the 10 total flowers and four purple flowers, as well as the implicit quantity of orange flowers (which students may immediately recognize as six flowers). Specific quantities allow students to connect the numbers they are working with to a context.

- **Will students understand the roles and relationships of the numbers and quantities in the problem?** If a problem contains many extraneous numbers, or multiple implicit quantities, a 3-Read may help students slow down and understand the information in the problem before they attempt to find an answer.

- **Have students been challenged to understand relationships between quantities in similar problems before, or to determine exactly what a similar problem was asking them to find?** If the relationships between quantities in a problem are complicated, using a 3-Read can help students better understand the structure of a problem and clarify what is being asked of them. Further, because the strategy focuses attention on the quantities *in context* before students move into solving, they are more likely to understand the question they are trying to answer before they begin making calculations.

- **Is the complexity of the language in a problem or the specific nuances of language likely to get in the way of students being able to understand what the problem is asking? Would a purposeful second reading of the text of the problem enhance student understanding?** In situations where students might read a problem once and quickly move to using operations on the numbers they see, slowing down to spend time understanding the context can give students a frame through which to interpret the numbers. Attending to the details of language can help to shed light on the relationship between the situation itself and the mathematics. Often, a solid understanding of the context can help students see the mathematical structure of the relationships and clarify a reasonable solution path, as well as to evaluate whether an answer is reasonable.

Participation Quiz

Purpose: support productive small group work on mathematics

GENERAL DESCRIPTION:

A Participation Quiz is a strategy for giving students real-time, specific feedback about the quality of their groupwork and their mathematical discussions in small groups. Through documenting how students are interacting with each other and with the mathematics, a teacher may assess the skills students are developing and, at the same time, create for students a robust picture of the habits and actions that connect to productive mathematical discourse.

In a Participation Quiz, the teacher defines specific expectations for student interactions in small groups. The teacher then documents publicly (either on the board or using a document camera) observations about the mathematical reasoning and group process successes of each group in the class.

It is a "quiz" in the sense that it captures a snapshot of a student's skills for engaging in collaborative mathematics. The evidence collected can be used as a formative assessment and to give a student feedback about her growing capacity for mathematical discussion.

At the end of the lesson, the teacher reserves time for students to read the public notes about their own actions as well as those of other teams and reflects on the feedback with the class. This reflection can take different forms depending on the focus of the Participation Quiz and the age level of the students.

WHEN AND WHY IS THIS USEFUL?

A Participation Quiz is useful:

- For a lesson that is mathematically complex enough that students' best teamwork skills are essential for success. A Participation Quiz can help students to recognize the need for collaborative effort as students make sense of a mathematical challenge.
- When participation is uneven, to focus students on listening to the ideas and questions of each person in their team.
- In situations where students are seeing or valuing only a narrow set of skills, to extend their thinking about what it means to do and learn math.
- When students need to focus on a specific vital action or develop their capacity to engage in a particular mathematical practice.
- Throughout the year, at times when the class needs a vivid reminder of what powerful teamwork and mathematical discussion look, sound, and feel like — a reminder of how much they can learn from working very well together.
- At the start of the year, to begin teaching students about high-quality teamwork and mathematical talk.

WHAT CAN STUDENTS LEARN FROM THIS EXPERIENCE?

- the habits, ways of interacting, and ways of sharing mathematical thinking that lead to successful collaborative learning.
- what engaging in a specific Student Vital Action or Mathematical Practice looks, sounds, and feels like.
- a broader definition of the abilities, actions, and ways of thinking and asking questions that connect to being mathematically smart.
- the value of hearing the contributions, ideas, and questions of each person in their team.
- specific language and actions that they can use to help their team succeed mathematically.
- what it looks and sounds like to perform a specific team role.

Participation Quiz: Step-by-Step

Ready to try a Participation Quiz? Here are step-by-step instructions for structuring a Participation Quiz in a lesson.

1. Start by picking an activity or exploration that is conceptually essential to students' success in a particular unit of instruction. The task should be written in a way that, while clear in its intent, still requires students to read and talk together to figure out what to do. (See "Picking a Task," below.)
2. Make decisions about which group norms, Student Vital Actions, or Mathematical Practices you want your students to focus on (ex: support ideas with evidence, ask questions, talk only to classmates within the group, helping doesn't mean giving answers, no one gets left behind, and so on). You may want to choose areas of emphasis that your students have been working on already, or you may want to use a Participation Quiz to focus your students more clearly on a groupwork norm or Vital Action that is still under-developed.
3. Announce to your class that the day's lesson will be structured as a Participation Quiz. Be clear about what you are looking for – what you expect to see and what you expect to hear. Show students how you will keep track of their work (such as on the board, or projecting notes using the document camera). Many teachers choose to record only positive feedback to reinforce what behaviors are valued, while others record positive feedback in one color and negative feedback in another. For example:

TEAM 1 (all positive)

- Quick start! Facilitator reading immediately.
- "Does everyone understand the question?"
- Building on each other's ideas: "So if we start by … what if we then ….
- Explaining in the middle of the table – all 4 leaning in to see and discuss.
- "I think the graph is saying that _____ because ____…"
- ""Wait, explain that again."
  All four sticking together: "Are we all ready for the next question"

TEAM 2 (mixed)

- All four reading quietly – Make sure you discuss!
- "Ok, what should we do first?"
- Talking outside group – Task Manager please do your job!
- Making statements but without reasons – Tell WHY!
- All four working but in pairs. Facilitator says, "Let's compare strategies."
- "I think the answer is ___ because I see ____."
- One person blasting ahead.

TEAM 3 (all positive)

- "Who wants to read aloud?"
- "So what it's asking is…"
- "Can you repeat your idea?"
- "So it sounds like we agree we need to ask the teacher…"
- "Wait, let me try to say that again."
- "What do you think?"
- "Does anyone need time to write?"

TEAM 4 (K or 1st grade)

- Asking Questions: ?????
- Ten Frame: ✓✓✓✓
- Working Together: ☺☺

4. Allow time at the end of the lesson for students to read (or hear, for younger students) the feedback. Students should see (hear) feedback from all groups, to broaden their understanding of what the desired behaviors look and sound like. Highlight key quotes and behaviors that supported groups to learn well together.

Picking A Task

Not every task lends itself to the Participation Quiz structure. Here are some things to keep in mind when selecting a task for students to work on during a Participation Quiz.

- **Is the mathematics essential?** A Participation Quiz can signal to students that the mathematics they are tackling is important enough that it requires their best collective thinking. If the idea represented in the task is small, it will be difficult to engage in rich discussion around.

- **Does the task represent important learning for all students?** Tasks that are designed to target the understanding of one or two key individuals turn into tutoring sessions rather than meaningful teamwork. Such tasks do not necessarily engage the full group in discussion.

- **Does the task have an entry point that will allow students to work with their teams to get started, without teacher intervention?** Challenging tasks give students reason to collaborate, but can also require the teacher to circulate to encourage, question, or provide key information before teams can work more independently. When the teacher is moving from group to group to provide support, she will not be able to step back to observe and document student interactions. This is not to say that students need not function through the entire Participation Quiz without teacher support. Rather, it is helpful if the teacher can observe teams working together to make sense of the task and begin work for at least the first 5 minutes of the Participation Quiz.

- **Does the task provide authentic opportunities for students to share ideas and strategies?** Tasks that allow for (or even require) students to approach and solve problems in different ways provide real opportunities for students to share ideas as they explore different strategies. Similarly, tasks that involve multiple steps, justification, and or analyzing a misconception give students something to talk about. If the problem requires straightforward computation students may not need a team to arrive at an answer, and groupwork may be forced.

- **Is the task clear enough that students will understand what they are working on?** Frustration mounts when students cannot agree on what they are asked to do. Clear tasks allow student discussion and debate to center on the mathematics they are making sense of.

- **What aspects of the task allow for observing the specific group norms, Student Vital Actions, or Mathematical Practices that are the focus of the Participation Quiz?** If the class is working on using team roles to further discussion, does the task provide a meaningful part for each role to play? Could it be modified to do so? If the focus is on precise language, do the questions in the task necessitate that kind of talk?

Tips and Notes

- It is not necessary (or possible!) to quote or record everything. Snapshots and sentences will provide ample data for an effective groupwork debrief at the end of the lesson. This is also an ideal opportunity to catch low-status students being "smart" and effective group members. It is also important to show independent workers how thinking out loud and working with a team can deepen their own understanding and learning.
- Depending on class size, some teachers remain in the front of the room to observe teams, taking notes at the document camera or on the board. Others circulate and listen, taking notes on a clipboard that they periodically post publicly. Either way, *it is important that observation notes remain as visible to students as possible.*
- The purpose of the Participation Quiz is to give students feedback about their groupwork in a manner that supports their learning. In general, the more specific the feedback is, the better students are able to use it as a guide. Direct quotes are especially valuable, as are specific behaviors that you observe. Simply writing, "Great teamwork!" or "Excellent facilitation" does not help students (especially those in other teams) realize what specific behaviors are valuable.
- Because space is usually limited, it is helpful to develop some shorthand (such as RM for Resource Manager or TOT for talking outside the team) and to only record some portions of direct quotes (*"What if we..."* or *"I think it's ... because ..."*).
- Some teachers prefer to record only positive feedback, others record positive and negative (perhaps using different colors, to help students to differentiate them), and some use a "strengths and questions" format. You are encouraged to find the format that works best for you and your students.
- It is common for students to "overact" at first, deliberately saying and doing things that they think the teacher wants to hear. This is not necessarily a bad thing, particularly if the team responds to the overacting with the kind of teamwork desired. Over time, this tendency will diminish.
- It is very important to leave enough time to debrief the Participation Quiz notes with the class. It is useful if the notes about each of the teams help the teacher tell why the team was successful (or not).
- As teachers become more comfortable with Participation Quizzes, they can also use them to track the growth in individual students' ability to engage in productive teamwork. Comments can be used in one-on-one conversations with students to push their development as mathematical thinkers and productive collaborators.

Participation Quiz Planning Tools

The following pages include a blank template for planning a lesson with a Participation Quiz, as well as examples of completed templates for one elementary and one secondary math lesson.

The planning templates are intended to guide a teacher's thinking as s/he prepares to use a Participation Quiz in a lesson, including thinking through the essential mathematics of the task as well as the Student Vital Actions or mathematical practices the students will be expected to exhibit. Thinking through these two pieces is critical to the Participation Quiz, as they will become the criteria for success that students are working toward. Pre-planning allows the teacher to articulate clearly what s/he will be observing for and to communicate that to students.

The final piece of the planning tool includes lesson closure. This is often where the deepest learning takes place, as students and teacher together synthesize and make sense of the math experience, share learning take-aways, and identify any lingering questions or misconceptions students may have about the content. It is also the time where students hear or read feedback on their participation in the group work and are able to reflect on their roles as participants and learners. Giving forethought to this piece of the Participation Quiz prepares the teacher to respond in a more informed way to the particular student experience, and helps to insure that adequate time is reserved for bringing closure to the lesson.

As the Participation Quiz becomes part of a classroom routine with a group of students, the need for the planning tool might decrease slightly, as the particular structures may become habits for both teacher and students. At that point, teacher planning can focus more narrowly on identifying the key mathematics and participation targets for the Participation Quiz.

Participation Quiz Lesson Plan Template

Grade Level: **Unit:**
Task:
Core Math Ideas:

Students will work in teams of _____

Frame for the activity:

In this participation quiz, I am listening/watching for how you:

-
-
-
-

As you work, I am going to be taking notes on your groupwork. I will be writing down the things that I see and hear that are supporting your group to understand the math and each other's ideas. If you notice that I have not written anything about your group after a while, think about how you are working together and what might need to change to make sure each person is participating and the group is working together.

Student Work Time

Circulate and record notes, with a specific eye for

-
-
-
-
-

Lesson Closure:

How will you wrap up the mathematics of the lesson?
How will you share feedback on the Participation Quiz with students?

Participation Quiz Lesson Plan: Elementary Sample

Grade Level: 1st **Unit:** 4 (OUSD Core Curriculum)

Task: "Match My Number"
Core Math Ideas: Match a number to multiple representations of the same value.

Students will work in teams of 4

Frame for the activity: In groups of 3 or 4, students will sort number cards, first making sense of the quantities represented in multiple forms, then matching them to 4 other representations of the same amount. Each group will create a poster showing the equivalency between representations.

In this participation quiz, I am listening/watching for how you:

- Keep the work in the middle of the table so all can see and participate
- Ask each other questions until ideas make sense
- Take turns placing the cards
- Make sure everyone on the team has a chance to explain their thinking about at least one of the quantities

As you work, I am going to be taking notes on your groupwork. I will be writing publicly and sharing with you the things that I see and hear that are supporting your group to understand the math and each other's ideas. If you notice that I have not shared anything about your group after a while, think about how you are working together and what might need to change to make sure each person is participating and the group is working together.

Student Work Time

Circulate and record notes, with a specific eye for

- *Leaning in, reaching across to point to another's work, moving cards around table*
- *working in the middle of the table*
- *asking questions, like: How do you know they're the same? Can you explain why they go together?*
- *"Jessica hasn't had a turn yet. Let her do one"*
- *"What do you think?" "Does that make sense?" "Wait, what did you mean?"*
- *"Has everyone had a turn putting cards together?"*
- *"I think these two go together because this one shows 2 tens and 2 ones, and this one shows 20 + 2. They both equal 22"*
- *Taking turns talking and listening to each other*

Lesson Closure:

Hang all posters on the wall and take a gallery walk. Encourage partners to talk about what they see and what they notice about the way other students matched the cards. Allow 1 or 2 groups to share their process, and another group to offer feedback.

- *Read notes aloud to class. Highlight some of the specific things that I saw either with a summary statement about characteristics of each group, specific quotes, specific body language, etc. Try to find one positive from each team. Need not be specific to an individual.*
- *ASK: Was there anything you noticed about the work in your own team (even if not captured by the teacher) that helped you make sense of the math today?*

Participation Quiz Lesson Plan: Secondary Sample

Grade Level: Algebra **Unit:** 1

Task: Field Trip Fundraiser (Adapted from SpringBoard Algebra 1, Activity 1.5)

Core Math Ideas: Solve multi-step equations with algebraic methods. Solve equations with variables on both sides.

Start with a 3 Read as a whole class to make sense of the context the questions relate to (pg 33). Then have students work in trios or groups of four for #1-6, using a participation quiz.

Frame for the activity: Tell students: In this participation quiz, I am listening for how you:

- Give reasons to explain your thinking
- use different representations to make your ideas clear
- ask questions to understand other people's ideas
- share your work in the middle of the table so that everyone can see
- make sure each person in your team has a chance to share her/his ideas, and that each person understands the team conclusion before moving on to the next problem.

"As you work, I will be taking notes on your groupwork. I will be writing down the things that I see and hear that are supporting your group to understand the math and each other's ideas. If you notice that I have not written anything about your group after a while, think about how you are working together and what might need to change to make sure each person is participating and the group is working together."

Student Work Time *25 minutes*

Circulate and record notes, with a specific eye for

- *body language showing engagement (leaning in, reaching across to point to another's work, moving around table)*
- *working in the middle of the table*
- *individuals advocating for needs*
- *checking in to hear each person's ideas*
- *asking for reasons and restatement of reasons when needed*
- *restating explanations*
- *What do you think? Does that make sense? Wait, what did you mean there?*
- *"I think it will be _____ because _____"*
- *"Has everyone had time to write? Are we ready to read the next question?"*

Closure

Share solutions and representations to 3 or 4 questions. (3 min)

To wrap up Participation Quiz:

- *2 minutes for individuals to read notes from all teams*
- *Highlight some of the specific things that I saw either with a summary statement about characteristics of each group, specific quotes, specific body language, etc. Try to find one positive from each team. Need not be specific to an individual.*
- *ASK: Was there anything you noticed about the work in your own team (even if not captured by the teacher) that helped you make sense of the math today?*

Additional tools for Engaging Students in Learning-focused Talk

In addition to the 3 Signature Strategies for Mathematics, there are many tools that can be used across disciplines to help support students to articulate their thinking, expand on their ideas, make connections, listen to the ideas of their peers, construct and critique the reasonableness of arguments, and engage actively in the different forms of classroom discussion.

- **The Talk Move Map** provides a series of "talk moves" or teacher responses to various levels of student participation, all geared toward increasing student discussion. Some of the talk moves are aimed at helping students expand on important ideas and concepts, while others go after helping them identify flaws in their own and each others' thinking. All of these recommended moves promote student talk, and place the teacher in the position of skillful facilitator, guiding students as they engage in talk to deepen their understanding of content.

- The **Facilitation Guide for Math Class** suggests facilitation strategies for the teacher that promote student talk and support students in participating actively during whole class discussions. They provide opportunities for students to talk about their own and each other's reasoning in provocative ways, and are designed to foster a classroom culture of respectful interaction among students and teacher. The strategies take into consideration the social and emotional learning needs of the diverse groups of students and different learning modalities present in any classroom, and offer multiple modes for whole group response.

Talk Move Map

You pose a question to the class.

Then, what if...

A STUDENT GIVES A RESPONSE.

You think:

Huh?? I didn't understand that at all!

useful move:

▶ **Say more**

examples

Can you **say more about that**?

Could you **say that again**?

Can you **give an example** of what you mean?

So let me see if I understand. **Are you saying...?**

You think:

Gee, good point! Did everyone get that?

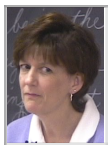
useful move:

▶ **Can someone rephrase or repeat that?**

examples

Can anybody **put that in their own words**?

Who thinks they **could repeat that**?



You think:

I think students got that, but I need to dig deeper into this student's thinking.

useful move:

▶ **Why do you think that?**

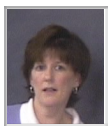
examples

What led you to **think about it that way**?

What's the **evidence** you used?

Can you **explain your reasoning** to us?

How did you **figure that out**?



You think:

Students heard this, but I want them to connect with this idea!

useful move:

▶ **What do other people think?**

examples

Who **agrees or disagrees, and why**?

Who wants to **add on** to what s/he just said?

What do you think about that idea?

Does anyone have a **different view**?

FACES BLANK. ONLY 2 HANDS RAISED.

You think:

I guess they need time to think!

▶ **Stop & Think** or **Stop & Jot** (60 seconds) *then*

▶ **Turn-and-talk** (60 seconds) *then*

▶ **Ask again!**

A STUDENT GIVES A RESPONSE THAT IS WRONG OR CONFUSED.

You think:

That's the wrong answer, but it might be very productive to discuss it!

Go back to the four moves to the left:

1. **Say more**
2. **Can someone rephrase that?**
3. **Why do you think that?**
4. **What do other people think?**

You think:

That's the wrong answer, and it's not going to take us anywhere!

▶ **Use your best judgment about how to move on.**

examples

Can you say that again?

Does anyone have a **different view**?

Well, actually, remember when we ... (give correction)

SEVERAL STUDENT RESPONSES ARE OFF TOPIC.

You think:

We've really gotten off track. Even though they're engaged, this isn't the question we're trying to consider!

▶ **Use your best judgment to get back on track.**

examples

Can you link this back to our question?

Can someone tell me how this fits in with our question?

Gee, what was our question? Let's recall where we're going...

SERP Strategic Education Research Partnership

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Facilitation Guide for Whole Group Instruction in Math Class

Consider the strategies below when planning to encourage and support math talk in a lesson. Select one or two strategies for a lesson:

- **Provide think time and wait time**
  - allow students time to think quietly for a minute before asking them to respond
  - after you've asked a question, wait at least 10 seconds before calling on anyone ("keep the answer in your head")

- **Vary the modes of response: give students options for how to respond to your questions, such as:**
  - "show it on your fingers"
  - "turn to a partner and whisper the answer"
  - "keep it in your head"
  - "raise a quiet hand"
  - "tell your partner your answer, then ask: 'do you agree or disagree?'"
  - "put your thumb to your chest when you have a strategy"
  - "all together…"
  - "once you have an answer, try to think of another way to solve it"
  - response cards (students hold up prepared cards with "true" "false"; numbers; "A", "B", "C", "D"; or another appropriate answer

- **Ask students to think and talk about each other's math**
  - "who solved it exactly the same way?"
  - "raise your hand if you understand exactly how ___ solved it"
  - "what do you think ___ was thinking when s/he solved it this way?"
  - "do you agree or disagree? Why?"

- **Encourage student-to-student conversations**
  - Ask each participant to call on the next speaker ("choose someone who has not had a turn to speak yet")
  - Ask: "does anyone have a question for ____?" and allow them to call on each other to ask and answer questions
  - Remind students to make eye contact with the person they ask or call on

- **Offer sentence frames on a poster or sentence strip**
  - "I agree with _____ because…"
  - "I disagree with _____ because…"
  - "I think ____ solved it like that because…"
  - "____'s idea is interesting because…"

LCI Math

Glossary of Terms

The terms below appear in OUSD Core Curriculum Units, Springboard mathematics materials, and in documents that describe curriculum, instruction and assessment in Oakland Unified School District.

3-Reads: A comprehension strategy that consists of reading the problem stem (the problem without a question) 3 times aloud with students, each time asking a different question:

- ✓ 1st Read: Comprehending the text
- ✓ 2nd Read: Comprehending the mathematics
- ✓ 3rd Read: Eliciting mathematical questions

3-Reads is one of three OUSD math signature pedagogies. Reference the Instructional Toolkit for a detailed description, "how to" information, and sample lesson plans.

Academic Discussion: OUSD's signature pedagogy across all content areas, grades TK – 12, happening through whole-class, small group and paired structures.

Quality academic discussions:

- ☐ are purposeful, sustained conversations about content
- ☐ require students to work together to co-construct knowledge and negotiate meaning in order to attain high levels of thinking and deep understanding about a topic
- ☐ are characterized by high student engagement and participation, as students discuss relevant topics that allow for multiple and diverse perspectives

Big Idea: A mathematically true statement that communicates the major understanding students should have as a result of completing a unit of study.

Engaging Every Learner: A section of the OUSD Core Curriculum Unit Plan that suggests instructional strategies that are task-specific to meet the diverse needs of every student. These strategies are targeted to meet specific needs and are not always necessary for every student.

Entry Task: An open-ended individual or group task that uncovers what students understand about the Big Idea in the unit and in what areas the students need more instruction on the supporting mathematics.

Essential Mathematics: The grade/course level mathematics that is essential to understanding the unit's Big Idea.

Essential Question: Anchors the unit with a question that students answer, justify, explain, and/or prove by the end of the unit.

Expanded Scope and Sequence: Provides teachers with an overview of suggested key learning experiences, instructional strategies, resources, and specifications for each OUSD Core Curriculum unit. Guides teachers in adapting their own units and meeting the tailored and specific needs of students.

Expert Task: An investigation or project that includes an individual product which provides students the opportunity to write (or talk) to construct viable arguments about the essential math using explanations of their findings and justifications.

Framing Student Learning: A section of the OUSD Core Curriculum Unit Plan that suggests how to plan class time for a specific learning experience.

- ✓ **Into:** Suggests how to introduce and/or get students started with the lesson or task.
- ✓ **Through:** Suggests how students do the math during the task. Specifically, how students are engaging in productive struggle and sense making building from their existing understanding and current knowledge.
- ✓ **Beyond:** Suggests how to bring the lesson to a close and identify learning to carry forward. Often supports students in making connections by suggesting specific questions for a whole class discussion to focus attention so that students leave with residue from the learning experience.

Formative Task: An open-ended group task that will deepen students' conceptual understanding of the math of the unit and provide important feedback about what students understand.

Key Learning Experience: Each unit contains learning experiences (Entry Task, Formative Task, Expert Task, Summative Task) critical for every student to engage with to build deep conceptual understanding about a unit's Big Idea. In addition, each one provides an opportunity to assess what students understand.

Learning Experience: The math task, lesson, or activity that will engage students in productive struggle and academic discussion to meet the math-learning goal of the lesson.

Learning Goal: The important mathematics that students should understand or wonder further about after completing the task. When included, the purpose statement in the Learning Goal describes how the experience ties in with the overall goals of the unit

Lesson Series: In the OUSD Core Curriculum each unit includes series of learning experiences between each unit key learning experience that continues to build new understanding of the Big Idea. Generally, each lesson series starts with a re-engagement task.

Misconceptions: Often truths that hold in some situations but not all, and interfere with students' ability to comprehend the mathematics when the "truth" no longer holds (see also: Prevailing Misconceptions).

Non-routine Problem: Problems and tasks requiring an abundance of mathematical action verbs – quantifying, generalizing, justifying, listing, modeling, getting precise, questioning, representing, visualizing, etc.

Number Talk: A 10 – 15 minute whole group *mental math* activity where students find the answer to a math problem in their heads, then share the strategies they used to find that answer. This strategy helps to develop quality student discourse in a whole class setting as students are encouraged to explain their thinking, justify their reasoning, and make sense of each other's strategies. Number Talk is one of three OUSD math signature pedagogies. Reference the Instructional Toolkit for a detailed description, "how to" information, and sample lesson plans.

Participation Quiz: A strategy for giving students real-time, specific feedback about the quality of their group-work and their mathematical discussions in small groups. Through documenting how students are interacting with each other and with the mathematics, a teacher may assess the skills students are developing and, at the same time, create for students a robust picture of the habits and actions that connect to productive mathematical discourse and learning. Participation Quiz is one of three OUSD math signature pedagogies. Reference the Instructional Toolkit for a detailed description, "how to" information, and sample lesson plans.

Prevailing Misconceptions: Statements that describe the common student over generalizations, beliefs, and misapplications of the content in the unit. For example, young students often learn that multiplication makes a number bigger. This can lead to significant misconceptions when students begin multiplying by fractions and decimals. Included on the OUSD Core Curriculum Year-Long Scope and Sequence documents.

Productive Struggle: Doing math with instruction that includes less step-by-step guidance, and more opportunities for true problem solving, and sense making.

Re-engagement Lesson: A task or lesson that promotes as much student discussion about misconceptions as possible by bringing out and examining important ideas. Moreover, this is an opportunity to confront and understand errors in the logic of student understanding.

Sample Assessment Problem: An example problem for each unit that represents problems included in the summative unit assessment, midyear exam and end of year exam. Included on the OUSD Core Curriculum Year-Long Scope and Sequence documents.

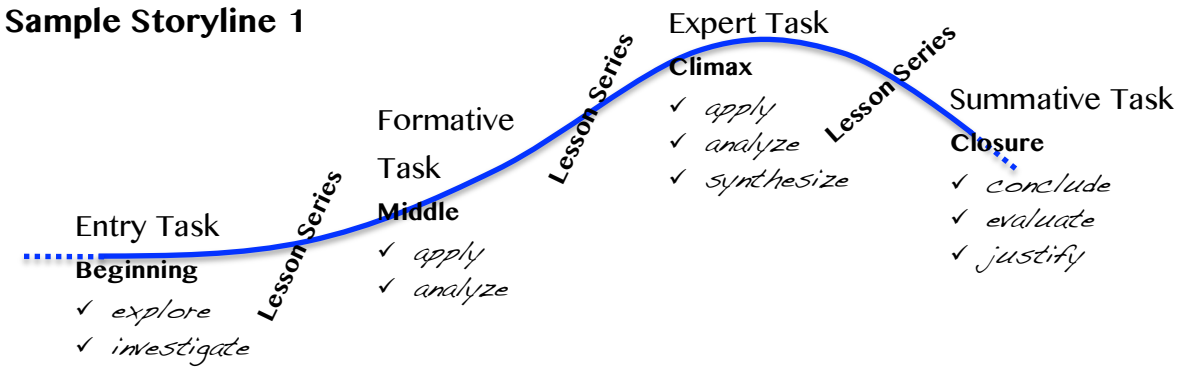
Summative Task: An individual assessment that gives information about what students know about the Big Idea (concepts and grade/course level strategies) at the end of the unit.

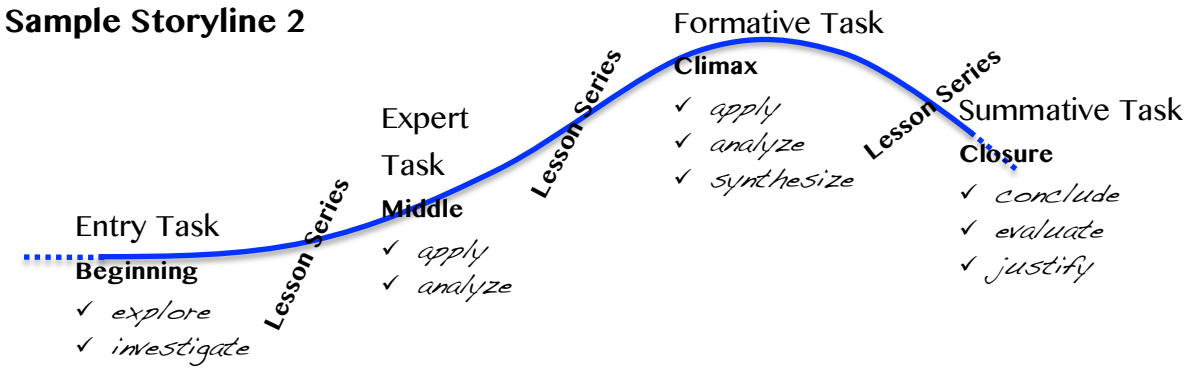
Supporting Mathematics: The mathematics that gives students access to the essential mathematics of the unit.

Tier II Academic Vocabulary: Tier Two words (what the Standards refer to as *general academic* words) are likely to appear in written texts rather than in everyday speech. They appear in all sorts of texts: information texts (words such as *relative*, *vary*, *formulate*, *specificity*, and *accumulate*), technical texts (*calibrate*, *itemize*, *periphery*), and literary texts (*misfortune*, *dignified*, *faltered*, *unabashedly*). Tier Two words often represent subtle or precise ways to say relatively simple things – *saunter* instead of *walk*, for example. Because Tier Two words are found across many types of texts, they are highly generalizable. (From CCSS-ELA, Appendix A)

Tier III Academic Vocabulary: Tier Three words (what the Standards refer to as domain-specific words) are specific to a domain or field of study (*lava*, *carburetor*, *legislature*, *circumference*, *aorta*) and key to understanding a new concept within a text. Because of their specificity and close ties to content knowledge, Tier Three words are far more common in informational texts than in literature. Recognized as new and "hard" words fro most readers (particularly student readers), they are often explicitly defined by the author of a text, repeatedly used, and otherwise heavily scaffolded (e.g., made a part of a glossary). (From CCSS-ELA, Appendix A)

Unit Storyline: Explains how the Big Idea of the OUSD Core Curriculum unit develops through the beginning, middle, climax, and end of the story of the unit. The verbs in the storyline reflect the conceptual evolution over the course of the unit.





Yearlong Scope and Sequence: Describes a year of study in the OUSD Core Curriculum. Guides the teacher on the required learning experiences, sequencing and suggested pacing, and assessments.

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Appendix

Professional Learning Opportunities
OUSD Core Curriculum
Field Trip Opportunities

Professional Learning Opportunities

LCI Math offers on-going professional learning opportunities for OUSD teachers to develop as practitioners in district-wide communities of practice and to deepen their understanding of math content and pedagogy.

Silicon Valley Math Initiative (SVMI) hosts monthly professional learning on instructional practices and assessment aligned to CCSS-M at the Tilden campus, September thru January. Sessions are free to OUSD teachers, and some support is available from LCI Math to pay for substitutes.

For more information about any of these offerings, please send email to the LCI math team at TeamMath@ousd.k12.ca.us.

OUSD Core Curriculum

The strategies included in this Toolkit can be used with any curricular materials. OUSD TeamMath recently completed a Core Curriculum Guide for Mathematics and units for each grade level and course aligned to CCSS-M. Those materials are available for use by all OUSD teachers via the OUSD Intranet at the following link:

https://sites.google.com/a/ousd.k12.ca.us/ousdcommoncorecurriculumguide/the-math-guide.

The curriculum production teams intentionally referenced and included the three key strategies in the Core Curriculum materials.

Field Trip Opportunities

The Bay Area provides access to an abundance of math-related field trip opportunities which can provide experiences that deepen students' engagement and help them draw connections between real life and mathematics. There is great value in extending mathematical experiences beyond the confines of a classroom, and LCI Math encourages schools and teachers to consider engaging in at least one math-related activity off school grounds. The following is a partial list of field trip destinations to consider:

Lawrence Hall of Science, Berkeley
New Bay Bridge tours (engineering)
Chabot Space and Science Center, Oakland
Local credit unit or bank
Exploratorium, San Francisco

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